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Holistic Perspectives on Instructional Design

Symbolizing and Communicating in Mathematics Classrooms: Perspectives on Discourse, Tools, and Instructional Design. Paul Cobb, Erna Yackel, and Kay McClain (Eds.). Mahwah, NJ: Lawrence Erlbaum, 1999.

Reviewed by Patrick W. Thompson
Vanderbilt University

"▽" represents a triangle. Or does it? It could represent perceived paths of light if drawn by a physicist explaining that the speed of light being the same in all reference frames implies that measures of time and distance are not. It could represent a jack o' lantern's eye if drawn by a child on Halloween. Or, it could represent a decomposition of morphisms if drawn by an algebraist in the midst of explaining commutative diagrams.

That we must say something about the conversation in which "▽" appears before we can say what it represents points to an issue at the heart of *Symbolizing and Communicating in Mathematics Classrooms*. As Erna Yackel says in her introduction,

> all of the contributors to this book take the position that it is the activity of symbolizing, not the symbols themselves, that should be the central focus of attention. Thus, rather than focusing on how individuals interpret symbols and use them to represent and convey meaning, the focus is on means for developing meaning and for recording and communicating that meaning. (p. 12)

The book examines this idea from a variety of viewpoints and traces out the idea's implication for instructional design. As an aside, I should point out that the book's title is crafted quite carefully. The book does address symbolizing, communicating, discourse, and tools, but it is ultimately about instructional design and theoretical foundations for it.

The book has chapters of three types: commentaries, theoretical considerations, and instructional design considerations. The commentaries are (a) an introduction to the book itself

(Yackel), (b) an introduction to the section entitled Theoretical Considerations (Paul Cobb), and (c) an overview of the book (Janet Bowers). Yackel provides context for the book by giving a history of its emergence from a symposium on discourse, tools, and instructional design and by explaining the editors' motive to situate issues of symbolization more closely with ideas of social contexts and away from individualistic perspectives in which symbols have referents and students' thinking is analyzed as if they never interact with anyone. Most readers will gain more by reading the Theoretical Considerations section first, and then returning to Cobb's introduction to it, as Cobb ties together the chapters in that section in nontrivial ways, so that the whole is greater than the sum of its parts. But one must be intimate with the parts to realize this. Bower's very useful discussion is of themes running through the individual chapters that might not be self-evident upon a first reading.

Theoretical Considerations

The Theoretical Considerations section collectively addresses a tough theoretical problem. If mathematical semiotic activity is thoroughly social and occurs only in discourse, then how do individuals ever move from a state of non-participation to a state of central participation? Anna Sfard's theme is anti-reference and anti-objectivist. She draws heavily on Peirce and de Saussere to make the claim that semiotic (sign-using) activity happens in discourse and has no direct parallel with a world of objects to which signs or symbols refer. Her chapter's title, Symbolizing Mathematics into Being, is quite suggestive of her thesis.

> Mathematical discourse and its objects are *mutually constitutive*: It is the discursive activity, including its continuous production of symbols, that creates the need for mathematical objects; and these are mathematical objects (or rather the object-mediated use of symbols) that, in turn, influence the discourse and push it in new directions. (p. 47)

Sfard does not address the issue of mechanisms by which individuals might become part of a discourse or how a discourse itself ever becomes constituted. Willi Dörfler, to some extent, and Bert van Oers, to a greater extent, do address this. Dörfler proposes two cultural objects, *prototypes* and *protocols*, that serve to guide individual's behavior but which do not enter the individual's consciousness. Prototypes are objects used by experts in their practice around which individuals can anchor behavior while having fragile understandings of what is being prototyped; protocols are behavioral patterns that individuals can emulate but which are captured in static displays of semiotic activity (e.g., a derivation). Dörfler does posit one essential individualistic construct that connects individual behavior with cultural objects of protocols and prototypes. It is the importance of individuals acting with an "as if" attitude. By acting "as if" the prototypes with which they act and the protocols which they follow make sense, when at the moment of acting they may not, the individual has an experience upon which he or she can reflect and, perhaps, construct meaning. Dörfler says,

> I emphasize that one of my purposes in stressing an action approach to finite mathematical concepts, via their protocols, is to counteract the prevailing static and figurative conception of school mathematics. In my view, the usual question of "what is the concept *xy*?" should be substituted, or at least complemented, by such questions as "which actions can be recorded and/or guided by the concept *xy*?" This brings the mathematics closer to the individual and makes it part of his or her experience beyond the purely linguistic realm. (p. 122)

Van Oers takes an explicitly Vygotskian approach to the problem of inducting students into the practice of mathematical symbolization. He draws heavily on three of Vygotsky's central psychological constructs (imitation, abbreviation, and predication) along with the main features of Vygotsky's sociohistorical psychology (interaction of practice-participating adult with practice-non-participating youngsters) and Leont'ev's and Gal'perin's activity theory to explain

the regeneration of symbolization as a cultural practice across generations. Van Oers stresses the fundamental role played by a teacher ("the participating semiotician") in raising issues of representing and recording so that they become part of classroom discourse. He describes students' attempted verbal expression as the beginning of a long process of removing themselves from direct experience and putting at a distance something in their understandings to which they eventually refer, and it is through a teacher's mentoring that students' attention is drawn to semiotic issues, and therefore appropriation of social practices.

Ricardo Nemirovsky and Stephen Monk focus on students' personal experience within the social context of symbolizing. Their theoretical focus is on curriculum design, and they instantiate their focus in an analysis of one student's (Lin's) interactions with a member of their research team (Tracey).

> In this chapter, we have attempted to articulate a conception of symbolizing as a creation of a lived-in space in which the absent is made present and ready at hand. In this lived-in space, the symbolizer fuses the present with the absent, as well as the symbols with the referents. In the second part, we argued that the lived-in space in which the symbolizing takes place is not controlled by deterministic forces (i.e., the use of certain symbols does not impose meaning on the symbolizer), but is pregnant with open trajectories that can be realized by trail-making — a manner of symbolizing that overlaps the experiential qualities of uncertainty and creativity. Finally, we described design as a creation of artifacts that develop as tacit points of contact between individuals and the cultural perspectives adopted and rejected by the designer. (pp. 213-214)

Nemirovsky and Monk attempt this through an analysis of seven minutes of interchange between Lin and Tracey as they discuss Lin's reconstruction of two bears' movements from a graph that records the bears' distances from a starting point with respect to the elapsed times of their trips.

The book's Theoretical Considerations section, as a whole, places issues of symbol use and symbolization squarely within a social context, proposing a theoretical foundation for addressing

how to design instruction and tools aimed at bringing students into a practice of symbolization. Some chapters contribute to this aim more directly than others. Sfard's chapter, while stating the strongest opposition to an objectivist approach to symbolization, suggests a new approach that some may interpret to be objectivist itself. This is that mathematical meaning and mathematical objects exist only in discourse, yet discourse exists only in the eyes of an observer. Van Oers's approach strikes a useful balance between individual action and social context, a balance that is essential for developing a theoretical foundation for mathematics instructional design. While Nemirovsky and Monk make interesting points regarding aspects of symbolization, their extensive analyses are based on interview dialog that is very difficult to follow. They argue against alternative interpretations of the dialog by explaining how the alternatives fail to fit events from classroom instruction, but they do not present those events as data. Said another way, Nemirovsky and Monk offer an intriguing theoretical perspective, although, in my reading, that perspective cannot be grounded compellingly in the data they present.

Instructional Design Issues

The book's second section is intended to build off the first. Yackel describes chapters in it as aiming to

> directly address issues of instructional design. In doing so, they discuss how semiotic processes can be exploited to proactively support students' development of the types of mathematical reasoning advocated in current reform documents. (p. 8)

Koeno Gravemeijer, Paul Cobb, Janet Bowers, and Joy Whitenack draw from two traditions in their approach to instructional design. The first is from Realistic Mathematics Education with its roots in Freudenthal's didactic analyses of mathematical ideas and in his notions of didactic phenomenology. The second is their adaptation of the sociocultural notion of practice to the special case of classroom mathematical practices. Their approach to design unites

their concern that students develop mathematical power with sociocultural perspectives on individual development as progressively more central participation in ongoing practices. They take their central design problem to be "how to support the development of both collective mathematical meanings and the understandings of individual students who contribute to their emergence" (p. 226).

Richard Lehrer, Leona Schauble, Susan Carpenter, and David Penner employ design tasks, with their intrinsic ill-structuredness, as a setting to examine the symbolizing and modeling in classroom activity and ways in which conventions and standards evolve. Their chapter provides a useful illustration of Sfard's and Dörfler's claim that mathematical meaning evolves through individuals' participation in discursive activity. Children's attempts to convey their understandings to others, either verbally or through inscriptions, also had the effect of transforming the understandings they were attempting to convey. This is consistent with van Oers's theoretical account of the transformative effect that social interaction among children and adults can have on children's understandings of what they take to be the objects of discourse.

Richard Lesh and Helen Doerr describe employing model-eliciting tasks as sites for students' local conceptual development. Their claim is that students' engagement with model-eliciting tasks propels them through a fairly well delineated series of modeling cycles, the product being a succession of increasingly sophisticated conceptions of the tasks and associated mathematical ideas. John Bransford, Linda Zech, Dan Schwarts, Brigid Barron, Nancy Vye, and The Cognition and Technology Group at Vanderbilt describe their current design theory as a natural evolution of their previous work in cognition and in technological innovation combined with an increasing awareness of significant mathematical tasks and of the importance of considering social context in students' motives for engaging in tasks meaningfully. One such

social context, in line with Lesh and Doerr's idea of model-eliciting tasks, is that students design reusable tools, and methods of employing them, with the intention that the products of their creative problem-solving activity will empower others."

The Instructional Design section has a strong synergy with chapters in the Theoretical Considerations section. Bowers points to this when she says

> The chapters illustrate Dewey's view of the critical relationship between community, communication, and the establishment of common meanings. From a theoretical point of view, this supports an epistemology in which learning is viewed as each individual's contribution to and participation in the negotiation of common practices. This theory informs pedagogical and design perspectives by suggesting that it is essential to anticipate the ongoing processes of collective reflection and generalization. … The implication from this reconceptualization is that although designers cannot know how any one particular student will interpret any given instructional activity, they can make curricular decisions based on predictions of how the collective discourse will support the negotiation of taken-as-shared meanings. (pp. 388-389)

As Bowers notes, the viewpoint that instructional design aims to influence classroom practices instead of influencing individual children has a clear advantage. This is that designers of instruction need not rely on psychological models of individual students to think about paths along which instruction might progress. This gives a clear practical advantage to the activity of design in that it allows designers to imagine a relatively unambiguous instructional goal—at least unambiguous in comparison to trying to imagine the effect an instructional sequence might have on individual students. Gravemeijer et al. design instruction so that certain norms of justification provide an impetus for children to participate productively in classroom discussions. They also design objects to be used by a teacher to ground particular ways of thinking about structuring numbers. Bransford et al. anticipate that students will come to approach tasks involving tools or their development assuming these tools will be used by persons other than themselves. Lehrer et

al. hope that design tasks will provide occasions for students to invent new inscriptions and to wrestle with matters of intention and convention.

At the same time that all these chapters exhibit a common reliance on the ideas of taken-as-shared meanings and classroom practice, I found lacking a careful analysis of certain constructs that are central to the work's development. It is important for authors to give careful consideration to the meanings they are attempting to convey when they present key constructs, because if readers are to employ these ideas, as opposed to merely adopting a terminology, they must have some confidence that they will reach judgements similar to the author's when applying the author's framework. For example, Gravemeijer et al. refer to the construct "practice" repeatedly throughout their chapter without giving an adequate explanation of what they mean by it. Readers should know what these authors believe a practice is, how one might determine that one exists, and how to distinguish among degrees of "practiceness." This would not be such a serious matter were it not for the fact that the authors' goal is to nurture sequences of practices. If their design methodology is to gain widespread use, others must be able to determine the degree to which a design's goals are met. Gravemeijer et al. also seem to use "practice" in somewhat incompatible ways. On one hand, they use "practice" colloquially, as customary or habitual action. (If you say of a group of men that they have a practice of tipping their hats upon meeting someone on the street, you imply that we should be surprised were one of them not to tip his hat.) At other times, they use "practice" as a collective construct— a pattern of behavior one discerns when viewing the group in total, though that behavior seems to imply nothing about the group's members beyond that they interact in ways that lead to this pattern's emergence. It would have been helpful if Gravemeijer et al. had communicated more clearly which meaning they had in mind at each use of "practice" because they seem to move

from one to another without saying so. Likewise, readers are likely to wish that Lesh and Doerr had more clearly explicated what they believe constitutes a "modeling cycle." They define a mathematical model and define modeling cycles that students go through in the context of model-eliciting tasks, however it is not clear how we can determine with any consensus where one cycle ends and another begins. In the same regard, I wonder what percentage of readers would say that students in Lesh and Doerr's study actually produced a mathematical model as judged by Lesh and Doerr's own criteria. These questions are not pedantic. Instead, they point to the need to be careful in the use of terminology. They are about creating a discourse among researchers, so that we have confidence that others will reach similar conclusions when working within an author's framework and when viewing the same data.

There is tremendous variation among these chapters' tacit expectations of teachers. At one extreme Lesh and Doerr describe a teacher's role as knowing to stay out of students' ways. At the other extreme Gravemeijer et al. describe a teacher as the one who employs power asymmetries to move classroom discussions in productive directions. In between, Bransford et al. take no stance on teachers' roles, and Lehrer et al. depict a teacher as a neutral sounding board. These different roles have tremendous implications for instructional design. I suspect the teacher Lesh and Doerr envision could not implement Gravemeijer et al.'s instruction effectively. Gravemeijer et al. seem to rely far more on teachers' ongoing activities as choreographer and conductor than do the others.

In my opinion, we might benefit from attempting to balance perspectives of an instructional designer and a classroom teacher. While it is sensible and legitimate for an instructional designer to be concerned with the shape of collective activity, and to count on using collective activity as an image of what he or she wants to have happen, teachers cannot do just this, for two reasons.

First, teachers must account for each child's progress, both to the child's parents and to the school system. Second, teachers must consider each child's readiness to participate in future social settings. They cannot promote or retain children collectively. They cannot recommend the class, collectively, for advanced programs or remedial instruction. Teachers might legitimately take collective activity as indexical of instructional progress, but they must also coordinate this with images of individual children's thinking.

Conclusion

Cobb, in his introduction to the Theoretical Considerations section, closes by saying

> [These chapters] do not fall into neat, oppositional camps. Instead, we have a network of commonalties and contrasts. This might be disturbing to those … who continue to argue that their own favored position constitutes an overarching viewpoint that gets things right independently of history, situation, and purpose. In my view, essentialist arguments of this type indicate a striking lack of self-awareness that contributes to the relative immaturity of mathematics education as a field of inquiry. (p. 35)

While it is true that these chapters do complement one another and offer rich contrasts, it is also true that there are competing points of view that are not represented, such as ideas of emergence (Holland, 1998; Schelling, 1978) or intersubjectivity (Steffe & Thompson, 2000; Thompson, 2000) that do not fit neatly with a Vygotskian or sociocultural tradition.

This book focuses on issues of instructional design in ways that are unique to the problems of mathematics education. It embeds problems of instructional design and symbolization within the larger context of human communication and socialization, and does so without losing sight of the fact that symbolization in mathematics is different than in other disciplines. I commend the editors and authors for their vision in addressing these issues of mathematics instructional design so broadly. I hope my remarks are accepted in the spirit of professional critique and commentary and do not convey the impression of having "got things right."

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